Trajectory Optimization for the Atlas/Centaur Launch Vehicle

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A method for finding solutions to complex trajectory optimization problems with detailed hardware and launch vehicle constraints is presented. The trajectory optimization problem is formulated as a constrained function minimization problem by modeling the control by a function of a finite number of parameters. Although any parametric control model may be selected, the natural choice of parameters is the actual parameters of the parametric guidance equations flown on the flight digital computer. A variation of the Method of Multipliers is presented for performing the constrained function minimization. A functional is proposed that allows incorporation of state variable inequality constraints. Finally, optimization of the HEAO-A direct-ascent trajectory is discussed. The HEAO-A mission demonstrates profile optimization to meet range safety, tracking, heating, structural integrity, and attitude rate constraints. The author's experience has shown his variation of the Method of Multipliers to be almost twice as fast as competing penalty function methods.

Introduction

COMPUTERS and automated computation algorithms have come to play a central role in the trajectory design and flight of current launch vehicles. Every step of design, checkout, and flight is now performed under automated computer control and supervision. The cost of inserting a payload on an interplanetary mission is currently well over $1000/lb. Thus, with computer and computation costs plummeting, it becomes cost-effective to use computers to improve reliability, reduce costs, and upgrade performance, especially for repetitive tasks. In this paper, we shall examine the HEAO-A trajectory optimization task. In that sense, this is not a theoretical paper; rather, it is a state-of-the-art expose.

The main portion of the paper is devoted to a discussion of the constrained optimization technique used, a method related to Hestenes' Method of Multipliers, but using various formulas to update the multipliers. To the best of the author's knowledge, no publications have appeared showing the applicability and advantages of multiplier methods to trajectory optimization problems. Before discussing the relationship of this method to other trajectory optimization methods, it is important to understand the guidance algorithms that implement the trajectories designed.

Atlas/Centaur Guidance

The Centaur Digital Computer Unit, a 16,000-word, 24-bit unit built by Teledyne, provides the guidance, navigation, telemetry, and event sequencing for all Atlas/Centaur and Titan/Centaur flights. The guidance algorithms are almost identical for both vehicles. Guidance can be thought of as being divided into two phases: 1) an open-loop phase, and 2) a closed-loop exoatmospheric guidance phase.

Attitude Control During Atmospheric Phase

Since wind shears may exist on launch day, structural integrity is a primary concern during the atmospheric phase of flight. The steering program for this phase of flight is chosen to fly near zero angle of attack through the region of high dynamic pressure to reduce bending loads caused by wind shears.[1,2]

Vehicle attitude during the atmospheric phase is modeled in the guidance computer as a function of altitude. For Atlas/Centaur, the altitude range from approximately 1400 ft through booster cutoff is divided into 11 altitude segments, and pitch and yaw attitude in each segment is modeled as a second-degree polynomial in altitude relative to the start of the segment. This modeling is illustrated in Fig. 1. Attitude is maintained vertical until 15 sec have elapsed from liftoff and guidance-predicted altitude is greater than the nominal design value.

Closed-Loop Guidance During Exoatmospheric Phase

Closed-loop guidance is initiated at an altitude of at least 80,000 ft and is usually delayed until after the booster package is jettisoned at approximately 2 min and 20 sec into flight. The Atlas/Centaur guidance equations fall in the category of precise parameterized guidance equations. A precise parameterized guidance scheme is one in which the steering law is assumed to be of a particular form and real-time integraion of the dynamic equations of motion is performed to determine the value of the guidance parameters that meet the desired end conditions. Near-optimal results are obtained by carefully selecting the functional forms to approximate the true optimal solution.

Functional forms used in the Centaur guidance equations can be obtained from the calculus of variations under some rather strong simplifying assumptions. If we assume a flat Earth, a constant gravitational acceleration, a constant thrust

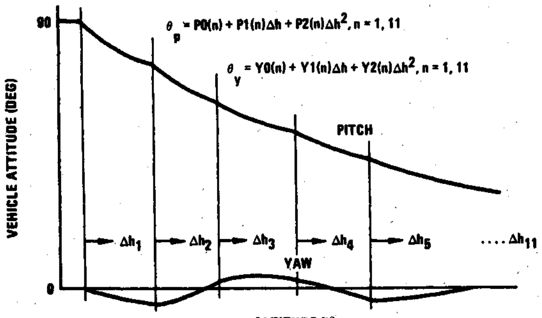

Fig. 1 Altitude is the independent parameter for booster wind-biased steering terms.

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and flow rate, no atmospheric drag, and the thrust vector contained in the instantaneous orbital plane, the calculus of variations leads to a bilinear tangent control law

$$\tan\theta = (C_1 t + C_2)/(C_3 t + C_4) \tag{1}$$

where θ is the pitch angle measured positive from the horizontal plane to a thrust vector above the plane and t is time from law initiation. The symbol C is used to represent constants throughout the paper.

If the terminal downrange distance is unconstrained, the transversality conditions indicate that $C_3 \equiv 0$, and Eq. (1) reduces to a simple linear tangent law

$$\tan\theta = (C_1/C_4)t + (C_2/C_4) = a + bt \tag{2}$$

Although none of the simplifying assumptions leading to this law actually is met in flight, remarkably good results are obtained by implementing the equation in a rotating coordinate system (tangential, normal, and radial) with θ always measured from the local horizontal.

A number of parametric control laws have been developed to control yaw steering out of the local orbital plane. The yaw steering angle measured in the local horizontal plane is shown in Fig. 2. A frequent launch vehicle targeting condition requires that a unit target vector, such as an outgoing hyperbolic asymptote $\mathbf{1}_a$, be contained in the orbital plane. One approach is to choose the yaw steering law to minimize the total velocity lost in the instantaneous orbital plane due to out-of-plane thrusting. An application of the calculus of variations leads to the following control law:

$$\sin\psi = C_5 (\mathbf{1}_t \cdot \mathbf{1}_a)/(V_t \cos\theta) \tag{3}$$

where V_t = the tangential component of velocity V, and the tangential direction is defined by unit vector $\mathbf{1}_t$. Bold symbols are used throughout the paper to represent vectors.

Although minimizing burn time to determine θ and minimizing velocity losses in the r-v plane to determine ψ do not necessarily result in a mathematically optimal trajectory, in practice, this approach has given good results for modest values of ψ. Other yaw steering laws under consideration are

$$\tan\psi = C_6 t + C_7 \tag{4}$$

$$\tan\psi = (C_8 t + C_9)/(C_{10} t + C_{11}) \tag{5}$$

These laws and variations of them are being considered for the HEAO mission. A complete discussion of the Centaur guidance equations appears in Refs. 3 and 4, while broader surveys of guidance techniques may be found in Refs. 5 and 6.

Parametric Trajectory Optimization

In this paper, we shall limit our attention to a method that explicitly recognizes the parametric modeling of the control law. In fact, parametric modeling is actually the case in most digital computer implementations. The more theoretical question of approximating the true continuous (albeit unimplementable) optimal control by a large number of parameters is discussed in greater detail in Ref. 7 and the references cited therein. A review of previous methods for optimizing trajectories and a discussion of the relative merits of previous methods compared with the parametric approach taken in this paper may be found in Ref. 8.

Problem Formulation

The proposed computational scheme has been implemented on Convair's existing detailed engineering simulation of the Atlas/Centaur launch vehicle. This was done to remove any doubt about the optimality of the solution due to simulation differences. The details of the problem formulation may be found in Ref. 8 or 9. The formulation allows for equality or inequality constraints on any function of the state variables at any point in the trajectory or on the integral of any function of the state variables. State variable inequality constraints are included in the formulation.

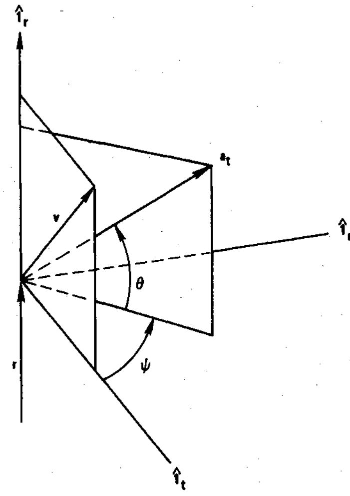

Fig. 2 Definition of thrust pointing angles θ and ψ in the *t-n-r* coordinate system.

Augmented Equation

The trajectory optimization problem may be cast into the following nonlinear programming formulation

$$\underset{x}{\text{minimize}}\ I(x) \tag{6}$$

subjected to

$$h(x) = 0 \tag{7}$$

$$g(x) \geq 0 \tag{8}$$

through the following definitions. Let the vector of all parameters subject to optimization be

$$x = [a_1, a_2, \ldots, a_w, d_1, d_2, \ldots, d_q] \tag{9}$$

where a_k is the vector of parameters associated with the parametric control model for u_k, $u_k(t) = V_k(a_k, t)$. The design variables d are variables to be optimized which are constant with respect to time (e.g., vacuum thrust, gross liftoff weight, or propellant mass fraction). Let the number of parameters in the x vector be NX.

If $\xi_s \geq 0$ is the sth state variable inequality constraint, then consider the transformation

$$T_s = \min_t [\xi_s(t)] \times \left[\int_{t_0}^{t_f} A\exp\{-B\max[\min(\xi_s(t), C_{\max}), C_{\min}]\}\mathrm{d}t \right] \tag{10}$$

where A, B, and $C_{\max}$ are positive constants, and $C_{\min}$ is a negative constant. Equation (10) is a functional that transforms a state variable inequality constraint function ξ_s into a number T_s such that T_s is negative if the constraint is violated and positive if the constraint is inviolate. A closer examination will show that the functional possesses other properties that are intuitively desirable.

The term preceding the integral is the minimum value of the state variable inequality constraint over all times during which the constraint is to be enforced. The integral term provides a measure of constraint encroachment. The integral term is always positive whether or not the state variable inequality constraint is violated. The integral term increases as violation is approached and continues to increase as violation worsens.

Figure 3 is a qualitative comparison of the behavior of the transformation functional and can be seen to yield results

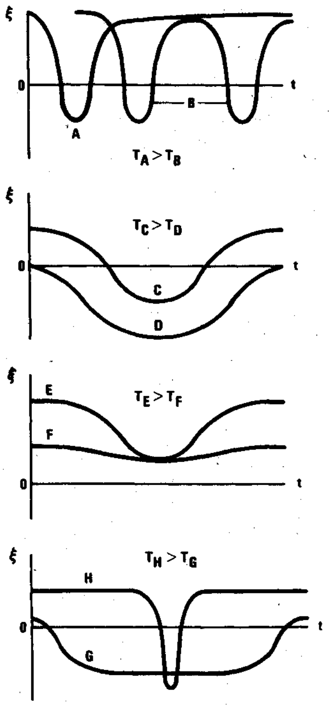

Fig. 3 Behavior of the state variable inequality constraint transformation.

compatible with our intuitive sense of the magnitude of a constraint violation. In the vicinity of $\xi=0$ this computational procedure is well behaved and compatible with the Method of Multipliers.

The constants A, B, $C_{\max}$, and $C_{\min}$ may be selected with some freedom and still maintain the desired properties. For the trajectory optimization problems solved here, the state variable inequality constraints were scaled so that a value of unity represents about 100 times the violation that would be considered tolerable. If this is done, $A=1$ and $B=1$ are satisfactory. $C_{\min}$ should be selected so that $|B^* C_{\min}|$ is less than the largest number that can be exponentiated safely without incurring an overflow. A value of -87 is safe for most present computers. $C_{\max}$ defines a distance on the positive side of the constraint beyond which the constraint is, in effect, ignored. We used $C_{\max}=10$. The transformed state variable inequality constraints T_s now may be considered to be elements of the inequality constraint vector $\boldsymbol{g}$. Let NH be the number of elements in vector $\boldsymbol{h}$ and NG be the number of elements in vector $\boldsymbol{g}$. With these definitions and transformations, the general control problem has been transformed into a nonlinear programming problem.

Method of Multipliers[10-18]

In this section, a variation of the Method of Multipliers is presented for solving the nonlinear programming problem of the previous section. These techniques form an augmented Lagrangian function by adding to the objective function both linear and quadratic functions of the constraints. Solution is sought by alternately minimizing the augmented function with respect to the independent variables and adjusting the linear coefficients (Lagrange multipliers) to satisfy the constraints.

These methods avoid the severe functional distortion inherent in the penalty function approaches (and, thus, the associated slowing of convergence and numerical problems) and admit the use of efficient quasi-Newton methods of function minimization. Moreover, since they do not require return to a feasible solution after each optimization step, as do feasible direction and gradient projection methods, computational load is reduced.

Consider the augmented function

$$J=I+\sum_{k=1}^{NH}\left[\lambda_k h_k+K_k h_k^2\right]+\sum_{k=1}^{NG}\left[\begin{array}{ll}\mu_k g_k+C_k g_k^2 & \text{if } Z_k\le 0\\ -\mu_k^2/4C_k & \text{if } Z_k>0\end{array}\right] \tag{11}$$

where

$$Z_k=\mu_k+2C_k g_k \tag{12}$$

and where

λ_k = equality constraint multipliers
μ_k = inequality constraint multipliers
K_k = equality constraint quadratic coefficients
C_k = inequality constraint quadratic coefficients

The computational procedure is outlined below.

1) Guess initial values for $\boldsymbol{x}$, $\boldsymbol{K}$, $\boldsymbol{C}$, $\boldsymbol{\lambda}$, and $\boldsymbol{\mu}$. Select tolerable errors for each of the constraints HTOL and GTOL. $\boldsymbol{K}=\boldsymbol{C}=\boldsymbol{1}$ and $\boldsymbol{\lambda}=\boldsymbol{\mu}=\boldsymbol{0}$ have proved to be satisfactory values. Select positive control tolerances $\epsilon_1<1$, $\epsilon_2<1$, ϵ_3, ϵ_4, and $\tau>1$. Set $i=1, j=1$. Evaluate $J_0(\boldsymbol{x})$.

2) Perform unconstrained function minimization of $J_j(\boldsymbol{x})$. After the first one-dimensional search ($i=1$) but before computing the gradient $\nabla \boldsymbol{J}^{(2)}$ at the new solution estimate, compute

$$\epsilon_5=\epsilon_1\|\nabla \boldsymbol{J}^{(1)}\|/N \tag{13}$$

where $\nabla \boldsymbol{J}^{(1)}$ is the gradient of J with respect to $\boldsymbol{x}^{(1)}$, N is the norm of the multiplier updates which would result if updating were done using the new solution estimate (see step 4d), and ϵ_1 is an input tolerance less than one (typical values range from 0.1 to 0.01). After one-dimensional search i but before computing gradient $i+1$, test for termination. If

$$\left.\begin{array}{l}\|\nabla \boldsymbol{J}^{(i)}\|\le\epsilon_5 N^{(i)}\\ |N^{(i)}-N^{(i-1)}|\le 0.01\epsilon_1 N^{(1)}\end{array}\right\}\quad \begin{array}{l}i\ge\max(2,NA)\\ \text{(see step 4c)}\end{array} \tag{14}$$

or

$$i\ge NX$$

go to step 3. Otherwise, set $i=i+1$ and continue the unconstrained function minimization. If, after any one-dimensional search, the function to be minimized is unbounded below, reset $\boldsymbol{x}$ to its initial value, reset $i=1$, and increase all values of $\boldsymbol{K}$ and $\boldsymbol{C}$ by a factor of τ. Repeat step 2.

3) Test for solution convergence. If

$$|[J_j-J_{j-1}]/J_j|\le\epsilon_2 \tag{15}$$

and

$$|h_k|\le \text{HTOL}_k \qquad (k=1,2,\ldots,NH) \tag{16}$$

$$g_k\ge -\text{GTOL}_k \qquad (k=1,2,\ldots,NG) \tag{17}$$

the current $\boldsymbol{x}$ is taken to be the solution, and iteration is terminated. Otherwise, go to step 4.

4) Update the Lagrange multipliers. A number of methods have been proposed for performing this step.[10-18] The method proposed here most closely resembles that of Ref. 14.

a) Determine the set of quasiactive inequaltiy constraints

$$I_a=\{k\,|Z_k<0,\quad k=1,\ldots,NG\} \tag{18}$$

where Z is given by Eq. (12).

b) Determine the maximum absolute value of all equality and quasiactive inequality constraints:

$$\text{AMAX}=\max[\,|g_k|,k\epsilon I_a;|h_k|,k=1,NH] \tag{19}$$

c) Let $NA=NH$ plus the number of elements in I_a. If NA is less than the number of elements in the $\boldsymbol{x}$ vector, NX, and $\text{AMAX}<\epsilon_3$, an input tolerance, go to step 4e. Otherwise, go to step 4d.

d) Update Lagrange multipliers by the gradient method[10]:

$$\delta_k=2K_k h_k \qquad (k=1,\ldots,NH) \tag{20}$$

$$\delta_{NH+k} = \left\{ \begin{array}{l} 2C_k g_k \text{ if } Z_k \leq 0 \\ -\mu_k \text{ if } Z_k > 0 \end{array} \right\} \quad (k=1,...,NG) \qquad (21)$$

$$\lambda_k = \lambda_k + \delta_k \quad (k=1,NH) \qquad (22)$$

$$\mu_k = \mu_k + \delta_{NH+k} \quad (k=1,NG) \qquad (23)$$

$$N = \|\delta\| \qquad (24)$$

Go to step 5.

e) Update the Lagrange multipliers by the least-squares method.[13] If Σ is the vector of quasiactive constraints, that is, $h_1,...,h_{NH}$, and g_k such that $k \in I_a$, and Λ is the vector of corresponding elements of λ and μ, then the new values for λ and μ are obtained by solving

$$\frac{\partial \Sigma}{\partial x} \frac{\partial I^T}{\partial x} = \frac{\partial \Sigma}{\partial x} \frac{\partial \Sigma^T}{\partial x} \Lambda \qquad (25)$$

for Λ. Values of μ_k for k not contained in I_a are set to zero.

5) Change the penalty constants as necessary.[11]

a) If $j = 1$, go to step 5d; otherwise go to step 5b.

b) If AMAX $< \epsilon_4$ AMAXOLD, go to step 5d; otherwise go to step 5c.

c) $K_i^{(j+1)} = \tau K_i^{(j)}$ if $|h_i| > \epsilon_4$ AMAXOLD, $(i=1,...,NH)$. $C_i^{(j+1)} = \tau C_i^{(j)}$ if $|g_i| > \epsilon_4$ AMAXOLD, $(i \in I_a)$.

d) AMAXOLD = AMAX.

6) Set $j = j+1$, $i = 1$, and, without reinitializing the current estimate of the Hessian of J with respect to x, return to step 2.

The computational procedure will automatically compute the values of the Lagrange multipliers λ and μ as the iteration converges. At the solution, the equality constraints and active inequality constraints are identically zero, and the multipliers corresponding to inactive inequality constraints are identically zero. Thus, the augmented function of Eq. (11) converges to the classical Lagrangian in the active constraints. As might be suspected, the active multipliers converge to the true Lagrange multipliers, and thus represent the actual sensitivities of the objective function J with respect to small variations in the constraints. Naturally, this is valuable information.

Although the procedure provides for automatic adjustment of the quadratic constant vectors $\boldsymbol{C}$ and $\boldsymbol{K}$, convergence is most rapid when minimal or no adjustment is required in the course of computation. This end is achieved in the computational implementation by automatically scaling the performance index I so as to be in the range from 1 to 10 and scaling the constraints by dividing each by 1000 times the corresponding HTOL or GTOL. Recall that HTOL and GTOL are tolerable errors in each of the constraints. If this scaling procedure is followed, initial values of $\boldsymbol{C} = \boldsymbol{K} = \mathbf{1}$ will require little if any adjustment in the course of computation. If the constraint scaling is poor, the procedures will compute reasonable values for the constant vectors $\boldsymbol{C}$ and $\boldsymbol{K}$. The algorithm provides only upward scaling; therefore, initial estimates for $\boldsymbol{C}$ and $\boldsymbol{K}$ should be chosen so that I dominates the augmented function. Good initial guesses for $\boldsymbol{C}$, $\boldsymbol{K}$, λ, and μ speed convergence; however, solution does not depend upon good guesses.

HEAO-A Trajectory Optimization

As an example of parametric trajectory optimization using the techniques outlined in the previous section, the trajectory for the first High-Energy Astronomical Observatory mission (HEAO-A) was optimized. The optimization was performed using a detailed launch vehicle simulation model. The simulation models an oblate rotating Earth, a fourth-order gravitational potential model, and detailed engineering

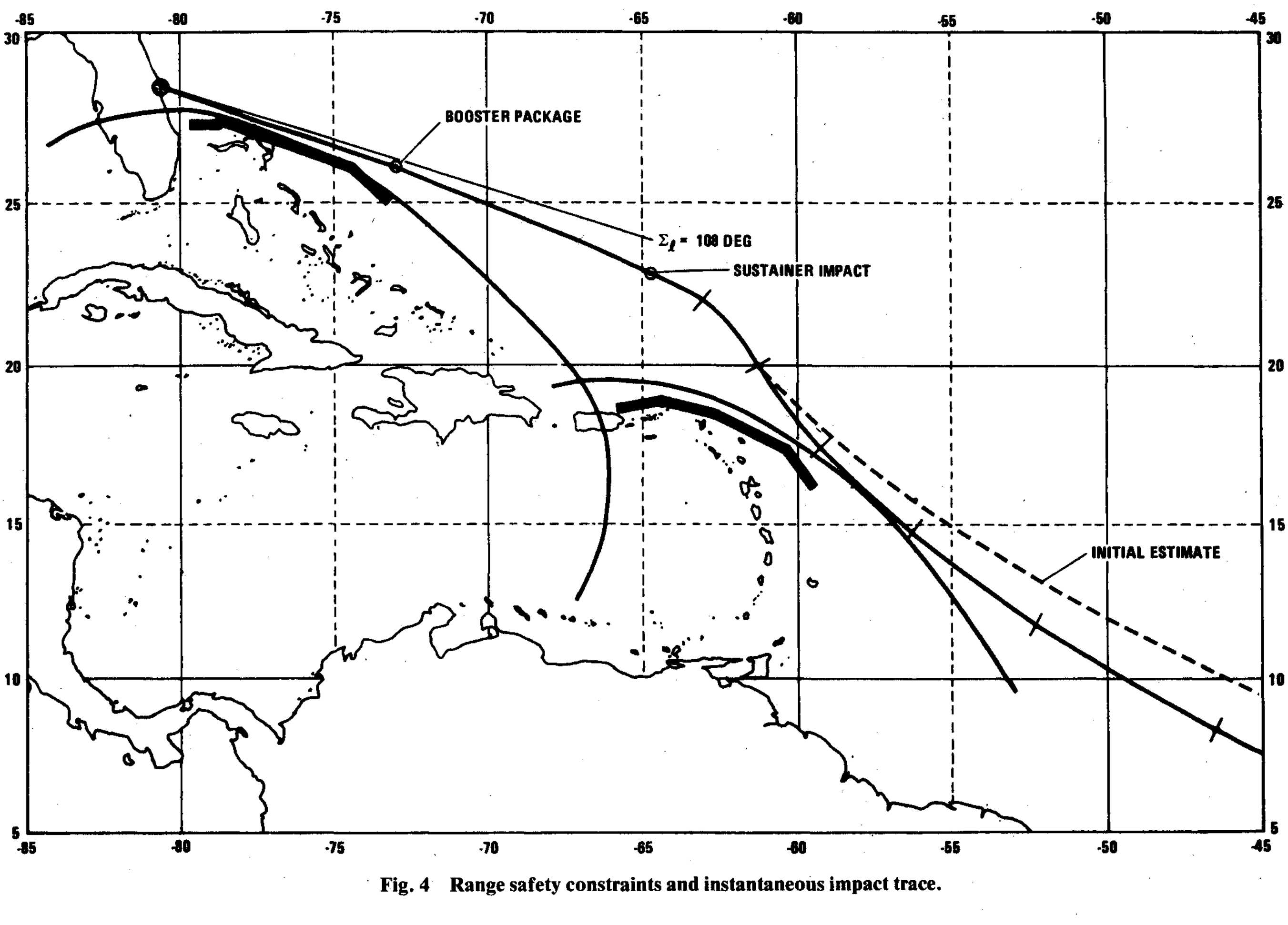

Fig. 4 Range safety constraints and instantaneous impact trace.

Table 1 Target condition errors (actual minus desired)

	Altitude (ft)	Velocity (fps)	Inertial flight path (deg)	Inclination (deg)	Range safety (deg)	Ascension view time (sec)
Initial	10,400	−93.70	1.56	−1.34	0.43	56.3
Final	977	− 1.98	−0.21	0.04	−0.003	−0.9

models for aerodynamics, propulsion, weights, centers of gravity, and winds.

The mission-peculiar specifications for the desired orbit are: altitude, 220.0 n.mi.; velocity, 25,145.8 fps; inertial flight-path angle, 0.0°; inclination, 22.75°; and tracking time from Ascension, >360 sec. Tracking time at Ascension Island is required to verify HEAO spacecraft requirements and is measured as the time during which the tracker elevation angle is greater than 5° above the horizon. In addition, the Centaur is to perform only a single burn. The inclination constraint, coupled with launch from Eastern Test Range and a single-burn Centaur, requires substantial yaw maneuvers to lower burnout latitude and meet the required inclination constraint. If range safety constraints were not imposed, a launch azimuth in the vicinity of 130° and a strong yaw-left steering program would be optimal. Range safety constraints, however, require a launch azimuth of no greater than 108°, and yaw steering to the right must be delayed to ensure that jettisonable items do not impact land masses.

In addition, it is desirable that the instantaneous impact trace (IIP) does not come within 30 miles of land mass, even during off-nominal flight. Figure 4 shows the range safety constraints in the area of interest. The heavy straight lines represent typical range limits on instantaneous impact point. This line cannot be violated without obtaining range permission on a case-by-case basis. The portions of ellipses shown represent IIP exclusion areas modeled for this example problem.

In addition to these constraints, the following launch vehicle structural integrity constraints are imposed: total attitude rate, ≤2 deg/sec; aerodynamic loading, αq, ≤250 psf-deg; aerodynamic loading at insulation panel jettison, αq, ≤4 psf-deg; post-nose-fairing-jettison dynamic pressure, q, ≤0.2 psf; and total aerodynamic heating indicator, $\int (qv)\mathrm{d}t$, $\leq 1\times 10^8$ lb/ft, where α is the total angle of attack, q is the dynamic pressure, and v is the velocity magnitude. The attitude rate is preprogrammed from launch through 80,000 ft and at predetermined times before and after jettisons and engine starts.

For design purposes, a body-rate turning program was simulated in the Atlas booster phase, with zero-angle-of-attack steering from 8000 to 80,000 ft, approximating the open-loop program design by a prelaunch system. Linear tangent steering [Eq. (2)] was used for the Atlas sustainer and Centaur phases. Yaw steering was controlled by guidance [Eq. (3)] during the Atlas sustainer phase and by guidance [Eq. (4)] during the Centaur phase. The following control parameters were selected: 1) a constant initial pitchover pitch rate from 16 sec after launch through 8000 ft; 2) a constant pitch rate from 80,000 ft to Atlas booster package jettison; 3) a constant yaw rate from 80,000 ft to Atlas booster package jettison; 4) the desired increment to the pitch attitude θ at the start of the Atlas sustainer phase; 5) the desired final pitch attitude θ at the end of the Centaur main engine burn; 6) the desired increment in yaw steering angle ψ at the start of the Atlas sustainer phase; 7) the duration of unpowered coast between sustainer jettison and Centaur prestart phase; 8) the desired increment in yaw steering angle ψ at the start of the Centaur main engine burn; 9) the desired final yaw steering angle ψ at Centaur engine burnout; and 10) the duration of the Centaur burn. Note that variable 6 determines constant C_5 of Eq. (3) with a given fixed target vector $\mathbf{1}_a$. Variables 4 and 5 determine constants a and b of Eq. (2), and variables 8 and 9 determine constants C_6 and C_7 of Eq. (4).

Optimization Results

In Figs. 5 and 6, the dashed lines show the initial control estimates. Initial values of the constraints are presented in Table 1. After 20 one-dimensional searches using Fletcher's VA10A algorithm,[19] the control had converged to the solid curves shown in Figs. 5 and 6 with corresponding constraint values shown in Table 1. The Centaur burn time was decreased by 15 sec. Figure 4 shows the initial and final trace of the instantaneous impact trajectory. The range safety constraint is clearly active and, in fact, drives the shape of the entire trajectory before the point of contact with the IIP ellipse. Rather than yawing south much earlier in the trajectory, very little yaw steering is done until the Centaur phase in anticipation of flying with a fixed steering law throughout the Centaur phase. A more optimal policy would allow changing from one parametric yaw steering law to another at the point of contact with the IIP ellipse. The first would be targeted toward the anticipated point of contact, and the second would be targeted to meet the inclination and view time constraints.

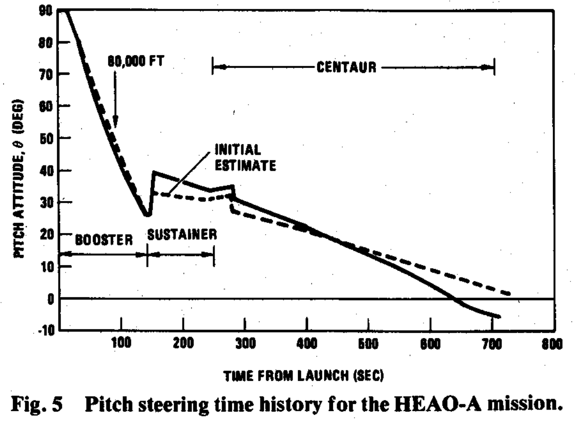

Fig. 5 Pitch steering time history for the HEAO-A mission.

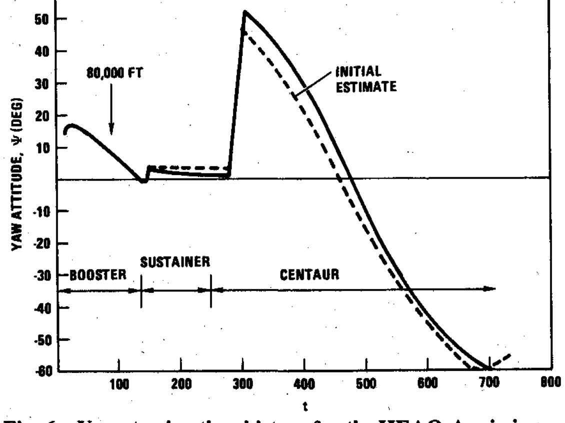

Fig. 6 Yaw steering time history for the HEAO-A mission.

Conclusions

A computational algorithm related to Hestenes' Method of Multipliers has been proposed for the solution of constrained optimization problems. The method shows promise of being computationally superior to feasible direction and penalty function methods on parametric problems. Through the use of state variable inequality constraint transformations and parametric control models, this paper has demonstrated how basic computational procedure can be extended to solve highly constrained optimal control problems, such as the HEAO-A mission.

Parametric approaches to trajectory optimization appear to be an attractive alternative to calculus of variations and method of gradients approaches, in light of decreasing computation costs, the complexity of launch vehicle simulation, and the actual parametric realization of "optimal" trajectory guidance. Although computational costs are higher for parametric methods, program maintenance and modification costs are reduced, and an extremely flexible program with rapid calendar time response is realized.

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References

[1]Brusch, R.G. and Reed, T.E., "Real-Time Launch Vehicle Steering Program Selection," *Journal of British Interplanetary Society,* Vol. 26, May 1973, pp. 279-290; also AIAA Paper 72-830, 1972.

[2]Swanson, D.C., "ADDJUST—An Automated System for Steering Centaur Launch Vehicles in Measured Winds," *Seventh Conference on Aerospace and Aeronautical Meteorology,* Melbourne, Fla., 16-19 Nov. 1976.

[3]Roberts, R.E., "Engineering Discussion of the Centaur D-1 Guidance Equations," Convair Aerospace, San Diego, Calif., GDCA-BKM71-007, Feb. 1971.

[4]Teren, F., "Explicit Guidance Equations for Multistage Booster Trajectories," NASA TN D-3189, Jan. 1966.

[5]Witcombe, D.W., "Present and Advanced Guidance Techniques," Aerospace Corp. Rept. No. TR-0073(3115)-1, El Segundo, Calif., Nov. 1971.

[6]Karacsony, P.J. and Cole, C.E., "Application of Parameterized Guidance," AIAA Paper 70-1006, Santa Barbara, Calif., 1970.

[7]Brusch, R.G. and Peltier, J.P., "Gradient Generation for Parametric Control Models," *Acta Astronautica,* Vol. 1, Nov. 1974, pp. 1453-1466.

[8]Brusch, R.G., "Trajectory Optimization for the Atlas/Centaur Launch Vehicle," *IEEE Conference on Decision and Control,* Clearwater Beach, Fla., Dec. 1-3, 1976.

[9]Brusch, R.G., "A Nonlinear Programming Approach to Space Shuttle Trajectory Optimization," *XXII Congress of the International Astronautical Federation,* Brussels, Belgium, Sept. 1971; also *Journal of Optimization Theory and Applications,* Vol. 13, Jan. 1974, pp. 94-118; also *Astronautical Research, 1971,* edited by L.G. Napolitano, P. Contensou, and W.F. Hilton, D. Reidel Publishing Co., Dordrecht, Holland, 1973, pp. 185-199.

[10]Hestenes, M.R., "Multiplier and Gradient Methods," *Journal of Optimization Theory and Applications,* Vol. 4, Nov. 1969, pp. 303-320.

[11]Powell, M.J.D., "A Method for Nonlinear Constraints in Minimization Problems," *Optimization,* edited by R. Fletcher, Academic Press, London, 1969.

[12]Hazelrigg, G.A., Brusch, R.G., and Sachs, W.L., "Optimal Space Trajectories with Delta Velocity Constraints," *Proceedings of the 21st International Astronautical Congress,* North Holland Publishing Co., Amsterdam, 1971, pp. 101-109.

[13]Haarhoff, P.C. and Buys, J.D., "A New Method for the Optimization of a Nonlinear Function Subject to Nonlinear Constraints," *The Computer Journal,* Vol. 13, May 1970, pp. 178-184.

[14]Buys, J.D., "Dual Algorithms for Constrained Optimization Problems," Ph.D. Dissertation, University of Leiden, Netherlands, June 1972.

[15]Brusch, R.G., "A Rapidly Convergent Method of Equality Constrained Function Minimization," *IEEE Conference on Decision and Control,* San Diego, Calif., Dec. 5-7, 1973.

[16]Kort, B.W. and Bertsekas, D.P., "Multiplier Methods for Convex Programming," *IEEE Conference on Decision and Control,* San Diego, Calif., Dec. 5-7, 1973.

[17]Martensson, K., "A New Approach to Constrained Function Optimization," *Journal of Optimization Theory and Applications,* Vol. 12, Dec. 1973, pp. 531-554.

[18]Brusch, R.G., "A Numerical Comparison of Several Multiplier Methods," *IEEE Conference on Decision and Control,* Phoenix, Ariz., Dec. 1974.

[19]Fletcher, R., "Fortran Subroutines for Minimization by Quasi-Newton Methods," Atomic Energy Research Establishment, Harwell, England, Rept. AERE-R 7225, 1972.